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# DESIGNING COLLECTION ROUTES THROUGH BANK BRANCHES

by

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## ABSTRACT

Most banks must send out vehicles on a regular basis to their branches in order to collect cash and negotiables left by depositors. The problem is modeled as a vehicle routing problem with constraints and objective terms specific to the particular banking context, such as penalties for lost interest. Two versions of the problem are studied: the deterministic case, and the stochastic case in which travel times are random. These two versions are modeled as integer mathematical programs, and solved by means of a heuristic procedure. Problems derived from a real situation (a Belgian bank network) are solved under different scenarios.


Keywords : Vehicle routing, integer stochastic programming.

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## RÉSUMÉ

La plupart des banques doivent envoyer à intervalles réguliers des véhicules recueillir les fonds déposés dans les différentes succursales. On modélise ce problème comme un problème de tournées de véhicules, avec des contraintes et un objectif propres au contexte bancaire. En particulier on doit tenir compte des pénalités engendrées par l'intérêt perdu. Deux versions du problème sont étudiées: le cas déterministe et le cas stochastique avec durées de trajet aléatoires. On modélise ces deux problèmes par la programmation mathématique en nombres entiers et on les résout à l'aide d'une procédure heuristique. Des problèmes dérivés d'une situation réelle (un réseau bancaire belge) sont ensuite résolus selon divers scénarios.


Mots-clés : Problèmes de tournées de véhicules, programmation stochastique en nombres entiers.

## 1. Introduction

Most banks must send out vehicles on a regular basis to their branches in order to collect cash and negotiables left by depositors and take them to a central office. This operation is usually carried out daily by the bank's own vehicles or by a contractor. The object of this paper is to analyze vehicle route planning in this context.

Our problem belongs to the family of vehicle routing problems (VRPs). These have been extensively studied in the operations research literature: see, for example, the surveys of Bodin, Golden, Assad and Ball [1], Christofides [2], Laporte and Nobert [12] as well as the book edited by Golden and Assad [7]. In essence, VRPs consist of designing minimum cost routes

(i) starting and ending at a common depot (central office);
(ii) visiting every customer (branch);
(iii) satisfying a number of side constraints.

Here, the only side constraint is that all vehicles must return to the central office before a preset deadline. In classical VRPs, routing cost is defined as a weighted sum of vehicle fixed costs and mileage costs. Our problem contains an additional cost component: money accumulated at a branch between the vehicle's visit and closing time is not collected until the next day and therefore loses one day's interest. This has a direct impact on the cost and shape of the solution since it may be preferable to delay as much as possible visits to branches producing a large amount of cash. Penalties associated with lost interest are directly taken into account in the objective function.

Vehicle capacity is always sufficiently large to make all collections and is not a binding factor. However, the total cash amount carried by a vehicle at any time should not exceed a given threshold imposed by insurance companies. But since this threshold is only rarely exceeded in practice, it is left out of our analysis.

Most VRP studies reported in the O.R. literature implicitly assume that all problem parameters are deterministic. This is clearly not always the case in practice. For example, the amounts collected at the various branches fluctuate from day to day and are random variables. Travel times are also random since they depend on traffic conditions. This affects cost since when a vehicle arrives at the central office after the deadline, all the money it carries is credited to the next day. We have chosen in this study to ignore the daily variations in the amounts collected at the branches, and to use instead their average values. Travel times are considered as deterministic in one version of the problem and as stochastic in another version. For recent references on stochastic VRPs, see (Laporte, Louveaux and Mercure [10], Laporte and Louveaux [9], and Dror, Laporte and Trudeau [6].

The remainder of this paper is organized as follows. In the next two sections, we describe two integer linear programming models for the problem: one for the deterministic case, and one for the stochastic case. We then show in Section 4 how these models can be solved by adapting the Clarke and Wright algorithm [3] originally designed for the classical VRP. In Section 5, computational tests and sensitivity analyses on both models are carried out on two data sets derived from the branch network of a bank operating in the French-speaking part of Belgium. The conclusion follows in Section 6.

## 2. Deterministic Model

The problem is defined on a graph $G=(V,A)$, where $V=\{0,1,\ldots,n\}$ is a set of vertices and A is a set of arcs. Vertex 0 represents the depot or central office, while the remaining vertices correspond to bank branches. With each arc $(i,j)\in A$ is associated a distance $d_{ij}$, a travel time $t_{ij}$ and a travel cost $c_{ij}$. The three matrices $(d_{ij})$, $(t_{ij})$ and $(c_{ij})$ satisfy the triangle inequality, and both $t_{ij}$ and $c_{ij}$ are linearly related to $d_{ij}$. The

service time at branch $i \in V$ is equal to $u_i$. Denote by T the maximal allowed duration of any vehicle route and by f the fixed daily cost of a vehicle. We assume the problem is defined on a daily basis, that everyday operations start at time 0 and, for the sake of simplicity, we ignore week-ends and statutory holidays. Branch i opens at time $r_i$, must be visited before time $v_i$ and closes at time $s_i > v_i$. Let $a_i$ denote the (average) daily sum collected at branch i. It is assumed that money accumulates at a constant rate over opening hours. Only money arriving at the central office before some cut-off time $T' > T$ is credited on the same day; otherwise, it loses one day's interest. The reason why $T' > T$ is that it makes little managerial sense to plan vehicle routes which arrive at the depot systematically after T'. Let $\alpha$ be the unit penalty for daily lost interest and finally, let M be an arbitrarily large constant.

Variables for this problem are defined as follows: $y_i$ is the arrival time of the vehicle at branch i, $x_{ij}$ is a binary variable equal to 1 if and only if arc (i,j) is used in the solution, and m is the number of vehicles. Note that $r_i$, $s_i$, $u_i$ and $v_i$ are known constants. The <u>deterministic model</u> is then

$$\text{minimize} \quad fm + \sum_{\substack{i,j=0 \\ i \neq j}}^{n} c_{ij} x_{ij} + \alpha \sum_{i=1}^{n} a_i \left[ \frac{\min\{T', v_i\} - y_i}{s_i - r_i} \right]$$

subject to

$$\sum_{i=1}^{n} x_{i0} = \sum_{j=1}^{n} x_{0j} = m \tag{1}$$

$$\sum_{\substack{i=1 \\ k \neq i}}^{n} x_{ik} = \sum_{\substack{j=1 \\ k \neq j}}^{n} x_{kj} = 1 \qquad (k=1,\ldots,n) \tag{2}$$

$$y_i - y_j + M x_{ij} \leq M - t_{ij} - u_i \qquad (i,j=1,\ldots,n;\ i \neq j) \tag{3}$$

$$\max\{t_{0i}, r_i\} \leq y_i \leq \min\{T - u_i - t_{i0},\ v_i\} \qquad (i=1,\ldots,n) \tag{4}$$

$$x_{ij} = 0 \text{ or } 1 \qquad (i,j=0,\ldots,n;\ i\neq j) \quad (5)$$

$$m \geq 1. \qquad (6)$$

In this formulation, the three terms of the objective function represent vehicle fixed costs, routing costs, and "indirect penalty" for lost interest, respectively. The factor within brackets is equal to the fraction of the money accumulated at branch i that is subject to penalty. If $v_i \geq T'$, all proceeds collected between the arrival of the vehicle (at time $y_i$) and the cut-off time T' lose one day's interest. Money accumulated between T' and $v_i$ also loses interest, but this is an unavoidable loss which cannot be reduced through optimization. If $v_i < T'$, the penalty applies to all proceeds received between $y_i$ and $v_i$.

Constraints (1) and (2) are degree constraints. Note that m can be treated as a variable, or as a constant when the size of the fleet is known. In this case, the first term of the objective function is unnecessary. Constraints (3) play three different roles: (i) they ensure that when j immediately follows i on a route, then $y_j \geq y_i + (u_i + t_{ij})$; (ii) they prevent the formation of subtours disconnected from the depot and (iii) they prohibit routes of total duration in excess of T. These constraints can be strengthened: using a lifting procedure, Desrochers and Laporte [5] show that it is valid to add an extra term equal to $(M - t_{ij} - t_{ji})\, x_{ji}$ to their left-hand side. Constraints (4) ensure that the arrival time of a vehicle at a branch occurs within opening hours, while allowing sufficient time for driving from and to the depot. Finally, constraints (5) and (6) impose integrality and non-negativity conditions on the variables.

## 3. Stochastic Model

In the stochastic model it is assumed that some arcs of A are congested with probability p. In practice, congestion is more likely to occur at rush hour, in downtown areas, or on roads leaving or entering major towns. Let $A' \subseteq A$ be the set of arcs subject to congestion and assume that for every arc $(i,j) \notin A'$, $t_{ij} = \beta_0 d_{ij}$ where $\beta_0$ is the inverse of vehicle speed; for every arc $(i,j) \in A'$, $t_{ij}$ is equal to $\beta_1 d_{ij}$ with probability 1-p and to $\beta_2 d_{ij}$ with probability p, where $\beta_2 > \beta_1$.

In such a context, it is important to make a distinction between two classes of decisions: (i) first stage decisions are those taken before knowledge of the actual "states of nature", i.e. congestion levels; (ii) second stage decisions are only taken once the outcome of the random variables is known.

In our problem, it makes sense to define first stage decisions as the a priori planned routes. Second stage decisions then correspond to the observation of the actual visiting times $y_i$ and to the computation of the associated costs. It is not realistic here to modify planned routes when congestion levels become known, since this would involve notifying the various branches of the new (expected) schedule and this could have an impact on the accounting operations at branch level.

The stochastic model can be derived from the deterministic model as follows. That part of the model which depends on first stage decisions only (m and the $x_{ij}$ variables) remains unchanged. This includes constraints (1), (2), (5) and (6) as well as the first two terms of the objective function. The modifications to the deterministic model will be presented in three steps: (i) constraints (3) and (4); (ii) the "indirect penalty" term of the objective function and (iii) a fourth term in the objective associated with possible late arrival at the depot.

### 3.1 Modified Constraints (3) and (4)

Even if travel times are stochastic, it is necessary to ensure that actual visiting times always take place within visiting hours. We also impose a constraint on the maximum expected duration of any vehicle route. These two constraints can be defined in terms of the expected travel time $\bar{t}_{ij}$ on arc (i,j) and of the expected arrival time $\bar{y}_i$ at branch i. Expected travel times are computed as

$$\bar{t}_{ij} = \begin{cases} \beta_0 \, d_{ij} & \text{if } (i,j) \notin A' \\ (1-p)\,\beta_1 \, d_{ij} + p\beta_2 \, d_{ij} & \text{if } (i,j) \in A'. \end{cases} \qquad (7)$$

Constraints on expected route duration can now be imposed by replacing $t_{ij}$ by $\bar{t}_{ij}$ and $y_i$ by $\bar{y}_i$ in (3).

We now wish to impose conditions on $\bar{y}_i$ so that no matter what the congestion levels are, actual visiting times are always feasible. In order to modify constraints (4) to that effect, define for every arc $(i,j) \in A'$: $\min(t_{ij}) = \beta_1 \, d_{ij}$ and $\max(t_{ij}) = \beta_2 \, d_{ij}$; for $(i,j) \in A'$, let $\min(t_{ij}) = \max(t_{ij}) = \beta_0 \, d_{ij}$. Then

$$\bar{t}_{ij}/\min(t_{ij}) = \begin{cases} (1-p) + p\beta_2/\beta_1 > 1 & \text{if } (i,j) \in A' \\ 1 & \text{if } (i,j) \notin A' \end{cases}$$

and

$$\bar{t}_{ij}/\max(t_{ij}) = \begin{cases} (1-p)\,\beta_1/\beta_2 + p < 1 & \text{if } (i,j) \in A' \\ 1 & \text{if } (i,j) \notin A'. \end{cases}$$

Similarly, define $\min(y_i)$ and $\max(y_i)$, the earliest and latest possible arrival times at branch i. Under worst case scenarios, the ratio $\bar{y}_i/\min(y_i)$ is equal to $(1-p)+p\beta_2/\beta_1$, and the ratio $\bar{y}_i/\max(y_i)$ is equal to $(1-p)\beta_1/\beta_2+p$, so that constraints (4) can be rewritten as

$$\max\{\bar{t}_{0i},\ r_i(1-p+p\beta_2/\beta_1)\} < \bar{y}_i < \min\{T-u_i-\bar{t}_{i0},\ v_i[(1-p)\beta_1/\beta_2 + p]\} \quad (i=1,\ldots,n). \qquad (4')$$

### 3.2 Modified indirect penalty term of the objective function

We are now interested in the expected indirect penalty at every branch i. Because of the linear relationship between this penalty and the arrival time $y_i$, it is valid to merely replace $y_i$ by its expected value $\bar{y}_i$ in the objective function.

### 3.3 "Direct penalty" for late arrival at the depot

Constraints (4') guarantee that even in worst case scenarios, all branches are always visited within opening hours and all vehicles arrive at the depot before time T. Since we have assumed $T < T'$, it follows that all money contained in these vehicles will be credited on the same day.

A less constraining scenario occurs when $T' < T < \min\{v_i\}$ and it is required that all collections take place within branch visiting hours. Here, return at the depot before T' is no longer mandatory. Then the right-hand side of (4') is simply replaced by $v_i((1-p)\ \beta_1/\beta_2 + p)$. Moreover, the possibility of losing one day's interest on the whole vehicle content must now be taken into account in the objective function. For vehicle route $\ell$, let $B_\ell$ denote the expected value of the vehicle content given that route duration exceeds T'. Then the objective function must include a fourth term equal to $\alpha \sum_\ell B_\ell$.

Since our model does not explicitly identify vehicles, i.e. setting $x_{ij} = 1$ does not indicate which vehicle travels on arc (i,j), the extra term just added in the objective function cannot be expressed in terms of the original variables of the model. Developing a more complex model to handle this case would be cumbersome and is quite unnecessary. Indeed, the new penalty term can be computed quite easily for any feasible solution. We now illustrate this computation for the case of mutually independent states.

Let $R_\ell$ be the set of arcs on route $\ell$: $R_\ell = \{(i_0, i_1), (i_1, i_2), \ldots, (i_{h-1}, i_h), (i_h, i_{h+1})\}$, where h depends on $\ell$ and $i_0 = i_{h+1} = 0$. Arc $(i,j) \in R_\ell$ is in state $\xi_{ij} = 0$ if it is not congested and in state $\xi_{ij} = 1$ otherwise. Let $\xi = (\xi_{ij})$ be a state vector associated with $R_\ell$ and define for every $(i,j) \in R_\ell$

$$t_{ij}^{\xi} = \begin{cases} \beta_0 \, d_{ij} & \text{if } (i,j) \notin A' \\ \beta_1 \, d_{ij} & \text{if } (i,j) \in A' \text{ and } \xi_{ij} = 0 \\ \beta_2 \, d_{ij} & \text{if } (i,j) \in A' \text{ and } \xi_{ij} = 1 \end{cases}$$

and $$p_{ij}^{\xi} = \begin{cases} 1 & \text{if } (i,j) \notin A' \\ 1-p & \text{if } (i,j) \in A' \text{ and } \xi_{ij} = 0 \\ p & \text{if } (i,j) \in A' \text{ and } \xi_{ij} = 1. \end{cases}$$

The amount collected at any branch $i_k$ is a function of $\xi$ and of the state $\xi'$ the previous day. In order to simplify matters, we assume that it is always equal to its expected value $a_{i_k}$. Now let $H_\ell = \left\{ \xi : \sum_{k=0}^{h} t_{i_k \, i_{k+1}}^{\xi} > T' \right\}$. Then

$$\bar{b}_\ell = \sum_{\xi \in H_\ell} \left( \prod_{k=0}^{h} p_{i_k \, i_{k+1}}^{\xi} \right) \left( \sum_{k=1}^{h} a_{i_k} \right) . \qquad (8)$$

This summation extends over all states $\xi$ for which the duration of route $\ell$ exceeds T'. The first factor is the probability that state $\xi$ occurs, whereas the second factor is the average amount of money carried by the vehicle.

## 4. Algorithm

The deterministic model just described is a modified version of the vehicle routing with time windows for which a number of column generation

techniques have been developed (see, for example, Desrochers, Desrosiers and Solomon [4]). These algorithms perform rather well on tightly constrained moderate size problems, but less well on problems with large time windows, as is the case in our application. Similarly, our stochastic model is a particular version of VRPs with stochastic travel times (Laporte, Louveaux and Mercure [11]). Although it could conceivably be solved by means of the branch and cut procedure proposed by Laporte and Louveaux [9], the size of our application would in all likelihood make this approach computationally difficult. Instead, we have opted for a heuristic algorithm that can easily be adapted to the problem under consideration and is also robust and efficient, the Clarke and Wright algorithm. This algorithm was originally designed to minimize functions consisting of the sum of routing costs in VRPs, i.e. functions of the form $z = \sum_{\substack{i,j=1 \\ i \neq j}}^{n} c_{ij}x_{ij}$. It can be summarized as follows.

Step 1. For every arc $(i,j)$ such that $i,j>1$, compute the saving $s_{ij} = c_{i0} + c_{0j} - c_{ij}$.

Step 2. Initially create n routes, each including the depot and one vertex of $V \setminus \{0\}$.

Step 3. Consider two vehicle routes containing arcs $(i,0)$ and $(0,j)$ respectively. If $s_{ij} > 0$, tentatively merge these two routes by introducing arc $(i,j)$ and by deleting arcs $(i,0)$ and $(0,j)$. Implement the merge if the resulting route is feasible. Repeat this step until no further improvement is possible. Stop. ∎

This procedure implicitly ignores vehicle fixed costs and fleet size. Vehicle costs can easily be taken into account by adding a constant f to every $c_{0j}$ $(j > 1)$. Solutions with a fixed number of vehicles can be obtained by repeating Step 3 until the required number of routes has been reached, even if the savings become negative.

There exist several variants and implementations of this procedure (see Paessens [14]). We have implemented the "parallel" version of the algorithm which consists of selecting the two nodes i and j to be linked according to the largest saving that can be generated.

This simple method can easily be extended to more general objective functions that do not simply minimize routing costs, but include other terms as well (such as penalties or expected penalties for lost interest). This is the case of both our deterministic and our stochastic models. Consider two vehicle routes $\ell_1$ and $\ell_2$ including arcs (i,0) and (0,j) respectively, and the costs $z(\ell_1)$ and $z(\ell_2)$ of these routes. Merging $\ell_1$ and $\ell_2$ by introducing (i,j) and deleting (i,0) and (0,j) yields a route $\ell$ of cost $z(\ell)$. If $\ell$ is feasible and the saving $s_{ij} = z(\ell_1) + z(\ell_2) - z(\ell)$ is positive, the merge is then implemented. This method can be quite time consuming since it requires updating the savings at each iteration. Computations can be eased, however, by using appropriate data structures (Paessens [14]) and heap-sorting methods (Nelson, Nygard, Griffin and Schreve [13]). Both these features were included in our implementation of the algorithm. Our program was coded in Turbo Pascal 4 and implemented on an Olivetti M380 microcomputer.

## 5. Computational Results

The above algorithm was tested on two data sets derived from a practical problem, both for the deterministic and stochastic cases.

We used the branch network of a bank operating in the French-speaking part of Belgium, with headquarters at Namur (used as the "depot"). The number of branches is equal to 28 in the first data set and to 44 in the second case. The branch names for each problem are given in table 1. The distance matrix $(d_{ij})$ was computed using the charts compiled by Houet and Cleeren [8]. Travel costs $c_{ij}$ were estimated at 15 Belgian francs (BEF) per kilometer. The cost of using a vehicle is estimated to be in excess of 10 000 BEF per day.

Table 1: Branch locations

Depot : Namur

Branches common to 28 city and 44 city problems

| | | | |
|---|---|---|---|
| Anhée | Floreffe | Jodoigne | Profondeville |
| Auvelais | Forville | Marbais | Rhisnes |
| Bois-de-Villers | Fosses | Mettet | Saint-Servais |
| Bouge | Gembloux | Moustier | Salzinnes |
| Eghezée | Jambes | Namèche | Sombreffe |
| Erpent | Jauche | Orp | Tamines |
| Flawinne | Jemeppe | Perwez | Walhain-St-Paul |

Additional branches for 44 city problem

| | | | |
|---|---|---|---|
| Assesse | Bouillon | Hastière | La Roche |
| Barvaux | Ciney | Havelange | Marche |
| Beauraing | Dinant | Hotton | Rochefort |
| Bièvre | Forrières | Jemelle | Wellin |

All branches open at 9 a.m., close at 4 p.m. and must be visited before 2 p.m., so that $r_i = 0$, $s_i = 7$ and $v_i = 5$ for all i. Also the values of T and T' were equal to 5 hours. The service time at any branch is equal to 5 minutes. Data on the daily amounts collected at the various branches are confidential, so we used estimates based on typical values, by assuming that $a_i$ is proportional to the population of city i. The daily penalty factor for lost interest was taken as 10/365 percent or 0.000274.

In deterministic problems, we executed tests with two different speeds: 25 km/h and 40 km/h. In stochastic problems, we defined A' as the set of arcs incident to one of the following cities: Namur, Dinant, Gembloux and Rochefort in 44 city problems, and Namur, Bouge, Gembloux, in 28 city problems, so that $|A'|=176$ in the first case and $|A'|=84$ in the

second case. These cities were chosen since they are those around which congestion is more likely. The value of p was set at 0.3 and the values of $\beta_0$, $\beta_1$ and $\beta_2$ were:

$$\beta_0 = 1/25,\ \beta_1 = 1/28,\ \beta_2 = 1/19 \text{ or}$$
$$\beta_0 = 1/40,\ \beta_1 = 1/47,\ \beta_2 = 1/26.$$

The number m of vehicles was fixed at different values for various runs, so that their fixed costs were ignored. Comparing the costs of the solutions obtained for different values of m may help decide whether it is better to use a larger or a smaller number of vehicles.

The algorithm was first applied to deterministic problems with the following data.

1) $n = 28$, $m = 5,4,3$, $\beta_0 = 1/25$, $\alpha = 0.000274$;
2) $n = 28$, $m = 5,3$, $\beta_0 = 1/40$, $\alpha = 0.000274$;
3) $n = 28$, $m = 3,2$, $\beta_0 = 1/25$, $\alpha = 0$;
4) $n = 44$, $m = 6,5$, $\beta_0 = 1/40$, $\alpha = 0.000274$;
5) $n = 44$, $m = 5,4$, $\beta_0 = 1/40$, $\alpha = 0$.

The main results are reported in table 2. These provide a good indication on the best number of vehicles to use. For example, in the first test series, going from 5 to 4 vehicles generates routes of reasonable length while adding very little to the total cost. Our algorithm failed to generate a solution with 3 vehicles (however, when $\alpha$ was set equal to 0, a feasible solution was found – see test series #3).

Table 2: Computational results for deterministic problems

| Test series | n | m | $\beta_0$ | $\alpha$ | Duration of shortest route (minutes) | Duration of longest route (minutes) | Routing cost (BEF) | Lost interest due to early collections | Total cost (BEF) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 5 | 1/25 | 0.000274 | 36 | 273 | 4650 | 2485 | 7135 |
| | | 4 | | 0.000274 | 167 | 273 | 4650 | 2704 | 7354 |
| | | 3 | | 0.000274 | | >300 | | | |
| 2 | 28 | 5 | 1/40 | 0.000274 | 27 | 186 | 4635 | 1773 | 6408 |
| | | 3 | | 0.000274 | 144 | 238 | 4275 | 2845 | 7120 |
| 3 | 28 | 3 | 1/25 | 0 | 279 | 299 | 4590 | 0 | 4590 |
| | | 2 | | 0 | | >300 | | | |
| 4 | 44 | 6 | 1/40 | 0.000274 | 144 | 300 | 11340 | 4130 | 15470 |
| | | 5 | | 0.000274 | | >300 | | | |
| 5 | 44 | 5 | 1/40 | 0 | 173 | 300 | 10800 | 0 | 10800 |
| | | 4 | | 0 | | >300 | | | |

Two test series were also executed for stochastic problems:

1) $n = 28$, $m = 8,6,5$, $\beta_0 = 1/25$, $\beta_1 = 1/28$, $\beta_2 = 1/19$, $\alpha = 0.000274$;

2) $n = 44$, $m = 12,9,6$, $\beta_0 = 1/40$, $\beta_1 = 1/47$, $\beta_2 = 1/26$, $\alpha = 0.000274$.

The main results are summarized in table 3. As for deterministic problems, this table enables comparisons between solutions containing different numbers of vehicles. When n = 28, the best value seems to be 5. The longest route in this solution has an expected duration of 230 minutes and a zero probability of lasting more than 300 minutes. It has a slightly larger expected variable cost than the solution with 6 vehicles, but a lower expected total cost when vehicle fixed costs are taken into account.

Table 3: Computational results for stochastic problems

| Test series | n | m | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\alpha$ | Expected duration of shortest route (minutes) | Expected duration of longest route (minutes) | Expected routing cost (BEF) | Expected lost interest due to early collections (BEF) | Expected lost interest due to late arrival at the depot (BEF) | Expected total cost (BEF) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 8 | 1/25 | 1/28 | 1/19 | 0.000274 | 15 | 279 | 7660 | 2220 | 0 | 9880 |
| | | 6 | 1/25 | 1/28 | 1/19 | 0.000274 | 77 | 230 | 5340 | 2168 | 0 | 7508 |
| | | 5 | 1/25 | 1/28 | 1/19 | 0.000274 | 153 | 230 | 5310 | 2440 | 0 | 7750 |
| 2 | 44 | 12 | 1/40 | 1/47 | 1/26 | 0.000274 | 11 | 281 | 12975 | 3147 | 0 | 16122 |
| | | 9 | 1/40 | 1/47 | 1/26 | 0.000274 | 35 | 299 | 12099 | 3612 | 3178 | 18889 |
| | | 6 | 1/40 | 1/47 | 1/26 | 0.000274 | 132 | 264 | 11520 | 3815 | 3082 | 18417 |

When n = 44, route durations sometimes exceed 300 minutes (although their expected duration is always less than 300 minutes). In two scenarios, the expected penalty for lost interest due to late arrival at the depot is positive. Using 6 vehicles is an attractive solution, given the high fixed cost of a vehicle. Paradoxically, with 6 vehicles, the probability of late arrival at the depot is equal to 30% while when 9 vehicles are used, this probability is equal to 55%. The total expected penalty is also slightly smaller with m = 6 than with m = 9. These unexpected results are explained by the presence of fewer random arcs on one of the routes with 6 vehicles, by a smaller expected length for the longest route and by the fact that a smaller sum is collected on that route.

## 6. Conclusion

We have analyzed in this paper a complex practical routing problem involving standard routing costs, as well as penalties for lost interest. In one version of the problem, stochastic travel times are also taken into account and the objective is then to minimize the total expected cost. Because of the intrinsic difficulty of this problem, and given the fact that typical problems are of medium or large scale, we have opted for a heuristic solution approach based on the Clarke and Wright algorithm. Problems derived from a real situation (a Belgian bank network) were solved under different scenarios. The results confirm the robustness of the proposed approach and provide decision makers with sufficient information to choose between different alternatives.

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